Ph.D. Entrance
Mathematics
w.e.f 2025-2026

Unit 1

Analysis: Elementary set theory, finite, countable and uncountable sets, Real number system as a complete ordered field, Archimedean property, supremum, infimum. Sequences and series, convergence. Bolzano Weierstrass theorem, Heine Borel theorem. Continuity, uniform continuity, differentiability, mean value theorem. Sequences and series of functions, uniform convergence. Monotonic functions, types of discontinuity, functions of bounded variation, Functions of several variables, directional derivative, partial derivative, derivative as a linear transformation, inverse and implicit function theorems. Metric spaces, compactness, connectedness. Normed linear Spaces. Spaces of continuous functions as examples.

Complex Analysis: Cauchy-Riemann equations. Contour integral, Cauchy's theorem, Cauchy's integral formula, Liouville's theorem, Maximum modulus principle, Schwarz lemma, Open mapping theorem. Taylor series, Laurent series, calculus of residues. Conformal mappings, Mobius transformations.

Unit 2

Modern Algebra: Permutations, Fundamental theorem of arithmetic, divisibility in Z, congruences, Chinese Remainder Theorem, Euler's Ø- function, primitive roots. Groups, subgroups, normal subgroups, quotient groups, homomorphisms, cyclic groups, permutation groups, Cayley's theorem, class equations, Sylow theorems. Rings, ideals, prime and maximal ideals, quotient rings, unique factorization domain, principal ideal domain, Euclidean domain. Polynomial rings and irreducibility criteria. Fields, finite fields, field extensions.

Linear Algebra: Vector spaces, subspaces, linear dependence, basis, dimension, algebra of linear transformations. Algebra of matrices, rank and determinant of matrices, linear equations. Eigenvalues and eigenvectors, Cayley-Hamilton theorem. Matrix representation of linear transformations. Inner product spaces, orthonormal basis.

Unit 3:

Ordinary Differential Equations (ODEs): Existence and uniqueness of solutions of initial value problems for first order ordinary differential equations, singular solutions of first order ODEs, system of first order ODEs. General theory of homogenous and non-homogeneous linear ODEs, variation of parameters, Sturm-Liouville boundary value problem.

Partial Differential Equations (PDEs): Lagrange and Charpit methods for solving first order PDEs, Cauchy problem for first order PDEs. Classification of second order PDEs, General solution of higher order PDEs with constant coefficients, Method of separation of variables for Laplace, Heat and Wave equations.

Linear Integral Equations:

Linear integral equation of the first and second kind of Fredholm and Volterra type, Solutions with separable kernels. Characteristic numbers and eigenfunctions, resolvent kernel.

Unit 4:

Numerical Analysis: Numerical solutions of algebraic equations, Method of iteration and Newton-Raphson method, Rate of convergence, Solution of systems of linear algebraic equations using Gauss elimination and Gauss-Seidel methods, Finite differences, Lagrange, Hermite and spline interpolation, Numerical differentiation and integration, Numerical solutions of ODEs using Picard, Euler, modified Euler and Runge-Kutta method.

Calculus of Variations: Variation of a functional, Euler-Lagrange equation, Necessary and sufficient conditions for extrema. Variational methods for boundary value problems in ordinary and partial differential equations.

Classical Mechanics: Generalized coordinates, Lagrange's equations, Hamilton's canonical equations, Hamilton's principle and principle of least action.

Unit 5:

Differential Geometry: Basic concepts of tensor algebra, Christoffel's symbols and covariant differentiation, space curves, tangent normal, binormal, curvature and torsion of space curves, Serret-frenet formulae, surface, curvilinear equations o curve on the surface, Tangent and normal, family of surfaces.

Fluid Mechanics: Viscous and non-viscous fluid, equation of motion, equation of continuity, conservation of energy, Reynodd's number, parallel flow, shear flow.

Special Functions: Bessel's function, Legendre's function, Hypergeometric series, generalized hypergeometric series.

Suggested Readings:

1. R. G. Bartle & D. R. Sherbert, Introduction to Real Analysis, Wiley India.
2. S. Ponnusamy, Foundations of Complex Analysis, Narosa Publications.
3. J. A. Gallian, Contemporary Abstract Algebra, CRC Press.
4. S. Lipschutz & M. Lipson, Linear Algebra, Schaum's Outlines.
5. M. D. Raisinghania, Ordinary and Partial Differential Equations, S. Chand.
6. A. M. Wazwaz, Linear and Non-Linear Integral Equations, Springer.
7. S.S. Sastry, Introductory Methods of Numerical Analysis, PHI.
8. A. S. Gupta, Calculus of Variations with Applications, PHI.
9. H. Goldstien, Classical Mechanics, Narosa Publication.
10. W. Kuhnel, Differential Geometry, American Mathematical Society.
11. R. Hibbeler, Fluid Mechanics, Pearson
12. M. E. H. Ismail, Theory and Applications of Special Functions.